McGill 00-16
hep-ph/0006037

Radion Mass is Milli-eV in the Goldberger-Wise Stabilization Mechanism

James M. Cline and Hassan Firouzjahi
Physics Department, McGill University, Montréal, Québec, Canada H3A 2T8

Abstract

We point out a large correction to the radion kinetic term, within the Goldberger-Wise radion stabilization mechanism, which has heretofore been overlooked. The radion mass is reduced by this factor, and takes the seesaw form $m_\phi \sim \mathrm{TeV}^2/M_{\rm Planck}$. We find that $m_\phi \sim 10^{-3} - 10^{-1}$ eV, which can be close to the limit of sensitivity of present-day short-range gravitational force experiments. Moreover, the couplings of the radion are suppressed by the Planck scale, not the TeV scale. Such a light radion suffers from the usual cosmological moduli problems. Similar considerations may also affect other stabilization mechanisms which rely upon bulk matter fields.

1. Introduction. The Randall-Sundrum (RS) idea [1] for explaining the weak-scale hierarchy problem has garnered much attention from both the phenomenology and string-theory communities, providing a link between the two which is often absent. RS is a simple and elegant way of generating the TeV scale which characterizes the standard model from a set of fundamental scales which are of order the Planck mass (M_p). All that is needed is that the distance between a hidden and a visible sector brane be approximately $b = 37/M_p$ in a compact extra dimension, $y \in [0, 1]$. The warping of space in this extra dimension, by a factor e^{-kby}, translates the moderately large interbrane separation into the large hierarchy needed to explain the ratio TeV$/M_p$.

However the RS idea as originally proposed was incomplete due to the lack of any mechanism for stabilizing the brane separation, b. This was a modulus, corresponding to a massless particle, the radion, which would be ruled out because of its modification of gravity: the attractive force mediated by the radion would effectively increase Newton's constant at large distance scales. An attractive model for giving the radion a potential energy was proposed by Goldberger and Wise (GW) [2]; they introduced a bulk scalar field with different VEV's, v_0 and v_1, on the two branes. If the mass m of the scalar is small compared to the scale k which appears in the warp factor e^{-kby}, then it is possible to obtain the desired interbrane separation. One finds the relation $e^{-kb} \cong (v_1/v_0)^{4k^2/m^2}$.

In [2], there is a nontrivial vacuum solution for the bulk scalar field, $\psi(y)$, due to its VEV's on the branes located at $y = 0$ and 1. The potential energy for the radion, $V(b)$, is found by substituting this solution back into the bulk Lagrangian for ψ and integrating over the extra dimension. The point of the present paper is that to determine the mass of the radion, one must compute not only $V(b)$, but the correction $\Delta F(b)$ to the radion kinetic term, $\frac{1}{2}(F(b) + \Delta F(b))\dot{b}^2$, which arises from the stabilizing field. The (mass)^2 of the radion is given by $V''/(F + \Delta F)$ at the minimum of the potential. In previous work, the ΔF correction was neglected, but we will show that it is actually of order e^{2kb} greater than F in the region where $e^{kb} \sim 10^{16}$, the value needed to solve the hierarchy problem. The radion mass is therefore suppressed by an additional factor of 10^{-16}.

In the rest of the paper we will derive this result, and compute the radion mass in a simplified form of the GW model which was introduced in [3]. As will be discussed, such a light radion is on the verge of being ruled out by present-day Cavendish experiments, but is presently still viable. Like any other modulus, it has significant cosmological difficulties however. We remark upon other implications of the radion renormalization effect in the conclusions.

2. Review of Goldberger-Wise Mechanism. In a recent paper [3] we presented a simplified version of the GW model which we will recapitulate briefly here. It consists of a free bulk scalar field ψ, with interactions on the two branes at $y = 0$ and $y = 1$. Using the 5-D metric $ds^2 = a^2(dt^2 - d\vec{x}^2) - b^2\, dy^2$ with $a(y) = e^{-kb|y|}$ and $b =$ constant, the scalar field's 4-D Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\int_{-1}^{1} a^4 \left(a^{-2}\dot{\psi}^2 - b^{-2}(\partial_y\psi)^2 - m^2\psi^2\right) b\, dy$$

$$- \int_{-1}^{1} a^4 \left[m_0(\psi(y) - v_0)^2\delta(y) + m_1(\psi(y) - v_1)^2\delta(y-1)\right] dy, \tag{1}$$

The points y and $-y$ are identified with each other, and the field thus satisfies $\psi(y) = \psi(-y)$. The extra dimension is therefore an orbifold with fixed points at the brane positions $y = 0$ and $y = 1$. The equation of motion is

$$-\partial_t(b\,a^2\dot{\psi}) + b^{-1}\partial_y(a^4\,\partial_y\psi) - m^2a^4\psi \;=\; 2m_0\,a^4(\psi - v_0)|_0 + 2m_1\,a^4(\psi - v_1)|_1 \tag{2}$$

with the boundary conditions implied by the delta functions $\partial_y\psi(0) = bm_0(\psi(0) - v_0)$ and $\partial_y\psi(1) = -bm_1(\psi(1) - v_1)$. We will now consider b to have a fixed constant value, and look for a static solution to the equation of motion. This solution can be expressed as

$$\psi = a^{-2}\left(Aa^{-\nu} + Ba^{\nu}\right); \qquad \nu = \sqrt{4 + m^2/k^2} \equiv 2 + \epsilon, \tag{3}$$

where $A = (-C_1\hat{\phi}^{\nu} + C_2\hat{\phi}^2)\hat{\phi}^{\nu}/D(\hat{\phi})$, $B = (C_3\hat{\phi}^{-\nu} - C_4\hat{\phi}^2)\hat{\phi}^{\nu}/D(\hat{\phi})$, $\hat{\phi} = a(1) = e^{-kb}$, $C_1 = \hat{m}_0v_0(\hat{m}_1 - \epsilon)$, $C_2 = \hat{m}_1v_1(\hat{m}_0 + \epsilon)$, $C_3 = \hat{m}_0v_0(\hat{m}_1 + 4 + \epsilon)$, $C_4 = \hat{m}_1v_1(\hat{m}_0 - 4 - \epsilon)$, $D(\hat{\phi}) = (C_2C_3 - C_1C_4\hat{\phi}^{2\nu})/(\hat{m}_0\hat{m}_1v_0v_1)$, $\hat{m}_0 = m_0/k$, and $\hat{m}_1 = m_1/k$.

The above solution must be substituted back into the bulk scalar field action to find the resulting potential energy for b, or equivalently $\hat{\phi}$. The result is

$$\begin{aligned} V(\hat{\phi}) &= m_0v_0(v_0 - \psi_0) + \hat{\phi}^4m_1v_1(v_1 - \psi_1) \\ &= m_0v_0(v_0 - (A+B)) + \hat{\phi}^4m_1v_1(v_1 - \hat{\phi}^{-2}(A\hat{\phi}^{-\nu} + \hat{\phi}^{\nu}B)) \\ &= \Lambda\hat{\phi}^4\left((1 + \epsilon/4 - \epsilon/\hat{m}_1)\,\hat{\phi}^{2\epsilon} - 2\eta\,(1 + \epsilon/4)\,\hat{\phi}^{\epsilon} + \eta^2\right) \end{aligned} \tag{4}$$

where $\Lambda \cong 4kv_0^2(1 + \frac{\epsilon}{\hat{m}_0})^{-2}(1 + \frac{4}{\hat{m}_1} + \frac{\epsilon}{\hat{m}_1})^{-1}$ and $\eta = (1 + \frac{\epsilon}{\hat{m}_0})(v_1/v_0)$. This potential has a minimum near $\hat{\phi} \cong (v_1/v_0)^{1/\epsilon}$, so if $v_1/v_0 \cong 0.7$ and $\epsilon \cong 0.01$, for example, the hierarchy of $\hat{\phi} \cong 10^{-16}$ between the TeV and Planck scales can be naturally generated, without any fine tuning of the model's parameters.

If the only source of the radion's kinetic term was the action for 5-D gravity, then the canonically normalized radion field would be given by [4, 5]

$$\phi = f\hat{\phi} = \sqrt{6}M_p\hat{\phi} \tag{5}$$

where $M_p^2 = (8\pi G_N)^{-1}$. The effective 4-D action for the radion and gravity would be

$$S \;=\; \int d^4x\,\sqrt{-g}\left(\tfrac{1}{2}f^2\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - V(\hat{\phi})\right) + \frac{M^3}{2k}\int d^4x\,\sqrt{-g}\left(1 - \hat{\phi}^2\right)R. \tag{6}$$

Here M is the 5-D Planck mass, so that (6) implies the relation $M_p^2 = M^3/k$. The radion mass, found from evaluating $V''(\hat{\phi}) \equiv \frac{\partial^2}{\partial\phi^2}V = f^{-2}\frac{\partial^2}{\partial\hat{\phi}^2}V$ at the minimum of the potential, would be

$$V''|_{\rm min} = 4\frac{\Lambda}{f^2}\left(1 + \tfrac{\epsilon}{4} - \tfrac{\epsilon}{\hat{m}_1}\right)^{-1}\left(\sqrt{1 + \tfrac{4}{\hat{m}_1}} + 2\tfrac{\sqrt{\epsilon}}{\hat{m}_1}\right)\,\eta^2\,\hat{\phi}^2\,\epsilon^{3/2}, \tag{7}$$

The main point of this letter is that the radion kinetic term receives an important contribution from the bulk scalar field action, which makes the physical radion mass many orders of magnitude smaller than V''. To this we now turn.

3. Correction to Radion Kinetic Term. In the above solution for the bulk scalar field, it was imagined that b was held constant. To find the correction to the radion kinetic term, we would like to consider background configurations where $b(x^\mu)$ is slowly varying with the coordinates along the branes. Because of Lorentz invariance of the RS metric, it is sufficient to restrict ourselves to a time-dependent radius, $b(t)$.

If $b(t)$ varies slowly enough, then the bulk scalar will simply track it. That is, $\psi \cong \psi_0[b(t)]$, where $\psi_0[b]$ is the solution already found in (3). There will be corrections to this which are higher order in derivatives of b. We can write

$$\psi = \psi_0 + \delta\psi \tag{8}$$

To leading order in derivatives of b, the only new contribution to the radion action will come from the $(\dot{\psi}_0)^2$ term in the bulk scalar field action (1), which vanished for the static solution. By assumption, all time dependence of ψ_0 is due to that of b, hence

$$\dot{\psi}_0 = \dot{b}\frac{\partial \psi_0}{\partial b} \tag{9}$$

It is straightforward to substitute this, using the solution (3), into the kinetic part of the action (1). In the limit $\hat{\phi} \ll 1$, the single term proportional to the cofficient B (and not $\partial B/\partial b$) of eq. (3) dominates. The new contribution to the radion Lagrangian is

$$\Delta\mathcal{L}_\phi^{(1)} = \frac{1}{4k}\left(\frac{\epsilon B\dot{\phi}}{\phi \ln \hat{\phi}}\right)^2; \qquad B \cong \frac{v_0}{(1+\epsilon/\hat{m}_0)} \sim M_p^{3/2}. \tag{10}$$

(Remember that $\dot{\phi}^2 \to \partial_\mu\phi\partial^\mu\phi$ if we allow ϕ to have both space and time dependence.) The crucial point is that near the minimum of the radion potential, $\phi = f\hat{\phi} \sim 10^{-16}M_p \sim 1$ TeV, whereas all the other dimensionful parameters in $\Delta\mathcal{L}_\phi^{(1)}$ are of order M_p. Therefore the coefficient of $\dot{\phi}^2$ is of order 10^{32}. We should renormalize ϕ accordingly. The physical radion mass is therefore given not by (7), but rather

$$m_\phi^2 = 12k\left(\frac{M_p\hat{\phi}\ln\hat{\phi}}{\epsilon B}\right)^2 V''|_{\rm min} \sim (10^{-3}\ {\rm eV})^2. \tag{11}$$

We have checked the consistency of the adiabatic expansion used to obtain the result (11), by estimating the next higher order correction. This comes from solving for the perturbation $\delta\psi$ and substituting it back into the nonkinetic part of the bulk scalar field action. One can compute $\delta\psi$ perturbatively, using the eigenfunctions f_n and eigenvalues λ_n of the Laplacian operator in the extra dimension, $\mathcal{O}_y = b^{-2}a^2(-\partial_y^2 + 4kb\partial_y + b^2m^2)$ which have been worked out in [6] (although the boundary conditions are different in the present problem, this turns out to be unimportant.) The perturbation is found to be

$$\delta\psi(y) = -\sum_n \frac{f_n(y)}{\lambda_n}\int_{-1}^{1} dy' f_n(y') b^{-1}\partial_t(ba^2\dot{\psi}_0) \sim \dot{b}^2, \ddot{b} \tag{12}$$

showing that indeed $\delta\psi$ is higher order in derivatives. Keeping careful track of all the factors of e^{-kb}, one can find upon substitution of (12) back into the action (1) the additional correction

$$\Delta\mathcal{L}_\phi^{(2)} \sim \epsilon^2 b^3 v_0^2 \left(c_1\left(\frac{\dot{\phi}}{\phi}\right)^2 + c_2\frac{\ddot{\phi}}{\phi}\right)^2, \tag{13}$$

where c_i are constants of order 1 and $b^3 v_0^2 \sim 1$, since $b \sim 1/M_p$ (actually $37/M_p$, but this will not change our conclusions). If the radion mass is m_ϕ, then $\dot{\phi}/\phi \sim m_\phi$, and we see that $\Delta\mathcal{L}_\phi^{(2)} \sim m_\phi^4$. This is much smaller than $\Delta\mathcal{L}_\phi^{(1)} \sim M_p^2 m_\phi^2$, so the adiabatic expansion we have used is an extremely good approximation.

With the new radion kinetic term, ϕ is no longer the canonically normalized field. Instead, it is

$$\Phi = -cM_p \ln(-\ln\hat{\phi}) = -cM_p\ln(kb); \qquad S_{\rm kin} = \tfrac{1}{2}\dot{\Phi}^2, \tag{14}$$

where $c = \epsilon v_0/(\sqrt{2k}M_p) \sim \epsilon$. Notice that unlike ϕ, Φ ranges in $(-\infty,\infty)$ as b goes from ∞ to 0. The radion potential now has the form of a double exponential in Φ. In the interesting region of small $\hat{\phi}$, it is a good approximation to expand the first exponential around the value Φ_0 which minimizes the potential, so that it is a single exponential:

$$V(\tilde{\Phi}) \cong \tilde{\Lambda}e^{4\tilde{\Phi}}(e^{\epsilon\tilde{\Phi}} - 1 - \tfrac{\sqrt{\epsilon}}{2})(e^{\epsilon\tilde{\Phi}} - 1 + \tfrac{\sqrt{\epsilon}}{2}). \tag{15}$$

Here $\tilde{\Lambda} \sim \hat{\phi}^4\Lambda \sim (\text{TeV})^4$, $\tilde{\Phi} = -\ln\hat{\phi}(\Phi - \Phi_0)/(cM_p)$, and we have omitted terms of order ϵ. The potential in the region of the minimum is shown in figure 1. Not shown is the small positive barrier far to the left, which was discussed in ref. [3].

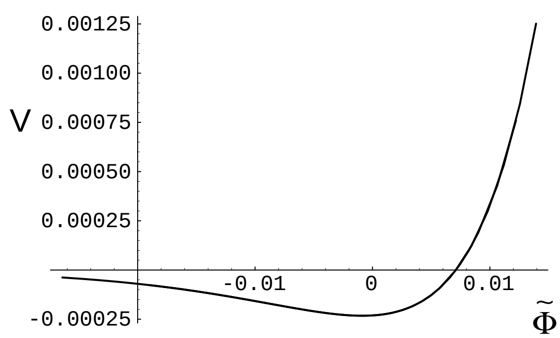

Figure 1: $V(\tilde{\Phi})$ in units of $\tilde{\Lambda} \sim (\text{TeV})^4$, versus $\tilde{\Phi}$, for the parameter values $\epsilon = 0.01$, $v_0/v_1 = 0.7$ and $\hat{m}_i \gg \epsilon$. The minimum value is $-\epsilon^{3/2}/4$.

4. Experimental and Cosmological Constraints. Interestingly, the scale 10^{-3} eV for the radion mass is close to but still above the limits implied by tests of the gravitational force at short distances. Of course these constraints depend not only on the radion mass but also on its couplings to matter. In [5] and [4], the radion is shown to couple to the trace of the stress energy tensor for a given particle; for instance it would couple to a massive fermion χ as

$$\mathcal{L}_{\rm int} = \frac{\phi}{\langle\phi\rangle} m_\chi \bar{\chi}\chi. \tag{16}$$

We recall that $\langle\phi\rangle = f\hat{\phi} \sim 1$ TeV. However, the canonically normalized radion field is $({\rm TeV}/M_p)\phi$, so the coupling is actually Planck-scale suppressed, and the radion couples with the same strength as gravity at very short distances. It gives rise to a force of the form $\alpha G_N m_1 m_2 e^{-m_\phi r}/r^2$ between two test masses, where $\alpha \sim 1$. The experimental limit on a scalar with gravitational strength couplings is $m_\phi > (1\text{ mm})^{-1} = 2\times 10^{-4}$ eV [7]. Improved experiments testing gravity down to 50 μm distances will be able to probe larger masses within a few years.

How large can the radion mass be? Putting together the above results and ignoring the correction factors which are close to unity,

$$m_\phi = 4\sqrt{2}\,\frac{k}{\epsilon^{1/4}}\,\frac{v_1}{v_0}\,\hat{\phi}^2 \ln\frac{1}{\hat{\phi}} \tag{17}$$

For consistency of the RS scenario, one needs $k < M_p$ to justify neglect of higher dimension gravitational operators; $v_1/v_0 \lesssim 1$, and ϵ could be as small as 0.1. Since $\hat{\phi} = {\rm TeV}/M_p$ and we take $M_p = (8\pi G_N)^{-1/2} = 2\times 10^{18}$ GeV, one might push m_ϕ as high as 0.05 eV, though significantly smaller values would perhaps be more natural.

More worrisome are the cosmological moduli problems presented by such a light scalar. (For a recent résumé of these problems, see ref. [8].) One does not expect the radion to be at the minimum of its shallow potential in the earliest moments of the universe. It seems more natural to start in the high-energy regime where $V \gtrsim ({\rm TeV})^4$ and $\Phi \gtrsim M_p$. If there is no damping of Φ's motion, it will roll past the minimum toward the region $\Phi \to -\infty$ where the branes become infinitely separated, leading to a visible brane-world where all particle masses are driven to zero. However it may be possible to get Φ to roll slowly into the region near the minimum; if $V'' < 9H^2$, where H is the Hubble parameter, the slow roll condition is fulfilled. If V has come to dominate the energy density of the universe then $H^2 = V/(3M_p^2)$, and the slow roll condition is equivalent to $c > 4/3$, where $c = \epsilon v_0/(\sqrt{2k}M_p)$, from eq. (14). Although c is naturally of order ϵ, one can imagine that v_0/k is large enough to compensate this. Nevertheless, the slow-roll regime will eventually give way to oscillations around the minimum of the potential, and these behave like nonrelativistic energy density of order $({\rm TeV})^4$, which will grossly overclose the universe. The lifetime of this coherently oscillating condensate is of order M_p^2/m_ϕ^3, many orders of magnitude larger than the age of the universe.

There have been many ideas for solving or ameliorating the moduli problems [9], including thermal inflation and the parametric resonance effect of reheating. Although a TeV scale radion mass is nicer from the cosmological point of view, the milli-eV possilibity is by no means ruled out.

5. Discussion. We have found that a 5-D bulk scalar field with a VEV of order $M_p^{3/2}$ makes a contribution of order $M_p^2(\dot{\phi}/\phi)^2$ to the radion kinetic term in the Randall-Sundrum scenario, which reduces the natural size of the physical radion mass to the milli-eV regime. There may be new sources of radion renormalization which we have overlooked, but it seems quite unlikely that they would be able to cancel the large one we have found with the precision that would be needed to render it harmless. Even if other stabilization mechanisms are found which do not have this difficulty taken by themselves, they will still get the same suppression of the radion mass if bulk fields with Planck scale VEV's exist. Although one could tune the VEV's to the TeV scale by hand, and thereby escape our conclusions, this would reintroduce the hierarchy problem that the RS idea was constructed to solve.

Even supposing such fields are absent, one should still consider possible new renormalizations of the radion wave function within a given mechanism. For example, ref. [10] showed that the Casimir effect from massless bulk fields gives a stabilizing potential which is qualitatively similar to that of GW. One should reconsider the problem in the situation where the brane separation b is slowly oscillating to see if new contributions to the energy density which are proportional to $\dot{b}^2$ arise. Ref. [10] found that the radion mass in this case is suppressed by $\hat{\phi}^{1/2} \sim 10^{-8}$ relative to the TeV scale, which is ruled out by considering the copious emission of radions from supernovae. However if the radion kinetic term were to receive a contribution of order $(10^7\dot{\phi})^2$, the couplings would be suppressed by 10^7 TeV, as needed to evade this bound. Coincidentally, the mass would be suppressed to the same small level (10^{-3} eV) as in the GW case.

One of our motivations for this work was the observation in [3] that the GW radion Lagrangian seems to suffer from an inherent pathology: the field ϕ is able to reach the value $\phi = 0$ in a finite amount of time, yet this represents the point of infinite brane separation. It is satisfying to notice that this problem is cured by the addition of the new kinetic term: $(M_p\dot{\phi}/\phi)^2$ damps the motion more and more as $\phi \to 0$. Or in terms of the canonical field Φ, the point $b = \infty$ corresponds to $\Phi = -\infty$, so again it takes an infinite time for the branes to separate to infinity.

However, our satisfaction is marred somewhat by the realization that, in the absence of any stabilization mechanism whatsoever, the radion is a free field with Lagrangian $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and ϕ can also reach zero in a finite time in this case. Therefore it seems there is still a puzzle as regards causality for propagation in the extra dimension. One observation is that the form $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ is only strictly valid in the Einstein frame, which was reached from the original string frame by a conformal transformation [4]. In the string frame where b is the true interbrane separation, there is mixing of the form $\dot{b}\dot{a}$ between the scale factor on the brane and that of the extra dimension. However this effect becomes negligible as $\phi \to 0$, so it is not able to damp the radion motion sufficiently to solve the problem.

Our previous work [3] is unfortunately invalidated by the present observation, since one must now consider the potential (15) rather than (4), and thermal transitions or tunneling to the true minimum of the radion potential may be suppressed if it is coherently moving in the direction $\Phi \to -\infty$. This analysis as well as that of [5] would be vindicated if the term $\dot{\psi}^2$ was simply absent from the bulk scalar field action. The objection that this would break 5-D Lorentz invariance is not so troubling, since the branes themselves do that. But the loss of causal propagation of signals in the extra dimension may be a greater concern, if indeed it has observable consequences for us who are confined on a brane.

We thank N. Arkani-Hamed, S. Jeon, M. Wise and G. Veneziano for their comments, and R. MacKenzie for helpful discussions about the moduli problem and Casimir effect.

References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221]; Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].

[2] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [hep-ph/9907447].

[3] J. M. Cline and H. Firouzjahi, hep-ph/0005235.

[4] C. Csaki, M. Graesser, L. Randall and J. Terning, hep-ph/9911406.

[5] W. D. Goldberger and M. B. Wise, Phys. Lett. **B475**, 275 (2000) [hep-ph/9911457].

[6] W. D. Goldberger and M. B. Wise, Phys. Rev. **D60**, 107505 (1999) [hep-ph/9907218].

[7] J. C. Long, H. W. Chan and J. C. Price, Nucl. Phys. **B539**, 23 (1999) [hep-ph/9805217].

[8] T. Banks, D. B. Kaplan and A. E. Nelson, Phys. Rev. **D49**, 779 (1994) [hep-ph/9308292].

[9] A. Linde, Phys. Rev. **D53**, 4129 (1996) [hep-th/9601083];
D. H. Lyth and E. D. Stewart, Phys. Rev. **D53**, 1784 (1996) [hep-ph/9510204];Phys. Rev. Lett. **75**, 201 (1995) [hep-ph/9502417].
G. Felder, L. Kofman and A. Linde, hep-ph/9909508.
G. Huey, P. J. Steinhardt, B. A. Ovrut and D. Waldram, Phys. Lett. **B476**, 379 (2000) [hep-th/0001112].

[10] J. Garriga, O. Pujolas and T. Tanaka, hep-th/0004109.